The Circle, or the Unreasonable Effectiveness of Mathematics

Ingmar de Boer, version 2.1, 18-12-2022

It is certainly true that in nature nothing miraculous exists, but in order to distinguish things we understand in terms of their causes from those of which the causes are unknown to us, we rightly call these miracles, not because they actually are, but because they so present themselves to us. [tr. IdB]

> *Tis wel waer datter inde Natuer niet wonderlick en is, nochtan tot onderſcheyt der dinghen die wy duer de oirſaken verſstaen, vande ghene welcker redenen ons onbetekent ſijn, ſoo gheuen wy deſe met recht de naem van wonder, niet dat ſijt eyghentlick ſijn, maer om dattet hem voor ons alſoo ghelaet.* [Simon Stevin, *Beghinselen der weeghconst*, Leyden, 1586]

Introduction

Let us suppose, we show a circle to a physicist and ask him to investigate it. He happens to be this rare scientist who has never seen a circle before, and is unbiased as he is not hindered by any prior knowledge about circles. We provide him with some tools, like a cord and a ruler, and ask him to examine larger and smaller circles in terms of their circumferences and diameters. For the first time in his life he is now confronted with the fact that the ratio of circumference and diameter of any circle seems to amount to one specific value. Every time he measures it, the ratio varies, but within only a few percent margin. He is surprised, overjoyed, and thinks this is nothing short of a miracle.

He then asks himself how this could be possible. After some serious thought he concludes that it must be the effectiveness of mathematical modelling which is the key to his success. There must be some "unreasonable effectiveness to mathematics", because of which its models are able to describe reality so accurately. Although our physicist has no special knowledge of circles, we hold him in high regard, and he is very reputable because of his many discoveries in other fields of physics. In response to his new insights, he decides to write an article in which he tries to incite other physicists to set up some preliminary research into the unreasonable effectiveness of mathematics. The physics community responds with excitement and expectation.

The Scope of Physics

As we know, Albert Einstein has remarked that "the only physical theories which we are willing to accept are the beautiful ones". Although obviously there is some truth to this, the beauty of mathematics itself is not of fundamental importance in the advancement of science. Physics and mathematics are often "beautiful", "miraculous" and seemingly "unreasonable", and that is exactly what makes these particular areas of interest so fascinating for many of us. We can be surprised or amazed by nature or by mathematics, while others may feel indifferent about the things we find miraculous. The terms surprised, amazed, etc. are of course purely subjective. The laws of nature are not essentially miraculous: they are just representations of certain relations we have found to exist between observables in specific circumstances in the world. In fact, the most important characteristic of these relations is that they are beyond the scope of the subjective.

Effectiveness of Language in General

The languages we humans use to communicate with each other, we call them natural languages, are all very effective means to describe the world around us. In some areas some languages may be more effective than others, but generally speaking it does not matter that much if we describe a certain situation, say, in Chinese or in English. For instance, there is never any doubt if Chinese children would be able to learn the same things and learn them as efficiently as English children, as a result of the capabilities of the language they have been growing up with. It may be our command of a language, our observations, our reasoning and many other things standing in our way when we describe the world, but, generally speaking, what is hindering us is not the language in which we try to describe things. We could say: "the world looks the same in any natural language." Vice versa, we could say that languages are apparently of an elastic nature, where "one language can fit all." The language is more effective if we can accurately encode and decode a larger part of our world with a more concise sentence and smaller vocabulary. This has something to do with the development of language itself, for example with the possibility of natural languages to form completely new sentences with a relatively small vocabulary, because of various grammatical mechanisms. Languages have evolved to very flexible tools, having solved for example in many different ways major syntactic problems. Ineffective mechanisms have died out in natural language, or have never come into being. In that respect, again, mathematics is not very different from natural language. The fact that we can describe the results of our experiments in mathematical or other terms is not at all miraculous: it is precisely what languages are good at.

The Language of Mathematics

All natural languages, also those which are historically unrelated, are all structured similarly in their deepest layers. In language we can see a reflection of how humans have modelled the world around them in the course of some thousands of years of their evolution. Languages are completely subject to their purpose, that is describing the specific surroundings of a social group. We can find however, only relations between observables in the outside world if we can describe them to ourselves in some form of language, explicitly or implicitly. Language and mind are in this sense not completely separate entities. We encode a model of these relations in the brain, if it were only to have the ability to reproduce them when asked, in the form of natural or mathematical language, for example when we are writing an article on it, or describing it to those who are interested. During the course of its evolution, the language of mathematics has become closely related to the way structures are stored in the brain. If, for the sake of the argument, we would go one step further, and equate mathematics with neural structures, it would be inappropriate to be surprised that the language we have used to store something in the brain, is able to effectively describe that same thing. Differently phrased: it was encoded in the brain in such a way that it can be decoded, so we should not be too surprised if that proves indeed to be the case. We could call it an example of "circular" reasoning.

Effectiveness Because of Natural Law, not Because of Language

The greater problem here, is attributing the accuracy of the relations to the effectiveness of the language in which we describe them. If we would measure the circumference and diameter of circle with a rope and a ruler, inaccuracies may arise due to elasticity of the cord, errors in placing or reading the ruler, etc. etc. If we look at the relation itself, the "natural law" behind the phenomenon, we see that it does not introduce error, because the underlying structure in nature will be that the precise ratio of circumference and diameter of a circle will amount to one specific value, which we call the mathematical constant π (pi). This regularity has already been discovered at some moment in the past, and we should expect it to be confirmed in future experiments. Recognising an element of surprise in hindsight would again be out of place here, because we have already shown long ago that a regularity exists. If there is a miracle here, it is perhaps that if a relation exists in objective reality, our subjective self is able to attain objective knowledge about it, be it often very fragmentary. This is in fact the miracle of the existence of physics itself. •